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A cosmological inflation model with inverse minimal and non-minimal coupling between scalar fields and curvature tensors

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ABSTRACT

This work reviews the cosmological inflation model involving inverted minimal and non-minimal interactions between the scalar field ϕ and its derivatives with the space curvature tensor. The de Sitter exponential expansion and the decaying scalar field conditions are also reviewed to move the model towards the inflationary condition, where as a generator of inflation, the scalar field must decay at the end of time. The scalar and tensor perturbation equations, their respective spectral indices, and the tensor-to-scalar ratio have been calculated to study the nonlinearity of the reviewed model. It is shown that the spectral indices and tensor-to-scalar ratio of the model are in good agreement with the observational data.

Keywords: Cosmological perturbations; inflation; scalar field

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INTRODUCTION

As a theory offering solutions to classical cosmological problems, inflation theory [1, 2] still leaves several issues related to its characteristics. Several studies involving measurements of the Cosmic Microwave Background (CMB), such as those conducted by COBE [3], the WMAP Collaboration [4], and the PLANCK Collaboration [5], provide support for an inflation theory in which the power spectrum of metric perturbations is comparable to CMB temperature fluctuations. The simplest inflation scenario is that a scalar field, called an inflaton, generates inflation when it is slow-rolled in an inflationary potential function $V(\phi)$. This scenario is called slow-roll inflation. However, the source of this inflation remains unknown.

Therefore, it is always possible to review cosmological inflation models involving scalar fields and their interactions with the curvature tensor. These range from modifications of the standard Theory of Relativity (TRU) to models with additional terms. One interesting model is the ADM formalism [6], which considers perturbation modes up to the second order via ADM variables. Furthermore, by selecting a gauge, the field perturbation can be substituted into a metric perturbation, so that the model can be analyzed solely as the evolution of the metric perturbation, and the results can then be compared with observational data.

Horndeski theory [7], one of several cosmological models involving scalar fields (often referred to as scalar-tensor theory), is the most feasible theory because its second-order field perturbation equations have solutions and do not contain the Ostrogradsky Ghost [8]. By choosing an arbitrary coefficient function, several well-known cosmological models such as minimal scalar field coupling [9], Brans-Dicke theory [10], Dilaton gravity [11], f(R) gravity [12], derivative coupling [13, 14], Gauss-Bonnet coupling [15, 16], and others can be derived. In this study, we explore the choice of coefficient functions in Horndeski theory to obtain a new inflationary model.

This paper is structured as follows: section 2 discusses the setup of the Horndeski Lagrangian model, incorporating the selected coefficient functions, and continues with an analysis of the non-perturbative background and its inflation solution. Section 3 calculates the spectral indices for scalar and tensor perturbations, along with the tensor-to-scalar ratio. Section 4 (final) provides conclusions and a discussion for further research.

SETUP MODEL

Horndeski's Lagrangian theory has the following form:

$$L = \sum_{i=2}^{5} L_i \qquad (1)$$

with,

$$L_1 = K(\phi, X) \qquad (2)$$

$$L_2 = -G_3(\phi, X)\Box\phi \qquad (3)$$

$$L_4 = G_4(\phi, X)R - 2G_{4X}(\phi, X)(\Box\phi)^2 + 2G_{4X}(\phi, X)\left[\phi^{;\mu\nu}\phi_{;\mu\nu}\right] \qquad (4)$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)\left(\phi_{;\mu\nu}\phi^{;\mu\nu}\right) + 2\left(\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}_{;\sigma}\right)\right] \qquad (5)$$

In this Lagrangian, the coefficient function (K, G_3, G_4, G_5) is a function of the scalar field ϕ and $X \equiv -\phi_{;\mu}\phi^{;\mu}/2$, the kinetic energy of the scalar field. The indexes in the coefficient functions indicate partial derivatives, $G_{Ix} \equiv \partial_{Gi}/\partial X$ and $G_{i\phi} \equiv \partial G_i/\partial\phi$, where $\Box \equiv \partial^\mu\,\partial_\mu = \partial^2/\partial t^2 - \nabla^2$ is the d'Alembert operator, R is the Ricci scalar, and $G_{\mu\nu}$ is the Einstein tensor.

In this study, the coefficient functions are considered as follows:

$$K = X$$
$$G_3 = 0$$
$$G_4 = \frac{M_{pl}^2}{2} - \frac{1}{2}\zeta\phi^2 \qquad (6)$$
$$G_5 = \xi\phi$$

where $M_{pl}^2 = 1/8\pi G \approx 1$. If we substitute the coefficient function (6) into the Lagrangian in Equation (2) to (5),

$$L = X + \left(\frac{1}{2} - \frac{1}{2}\zeta\phi^2\right)R + \xi\phi G_{\mu\nu}\phi^{;\mu\nu} \qquad (7)$$

The Lagrangian above provides a cosmological model with minimal and non-minimal coupling between the curvature tensor and the scalar field and its derivatives, where ζ and ξ are the coupling constants for the scalar field and its derivatives, respectively.

The space-time metric of the universe with cosmological perturbations can be written in the form,

$$ds^2 = -dt^2 + h_{ij}dx^i dx^j \qquad (8)$$

Considering the Maldacena gauge to improve the time and spatial reparameterization [19],

$$h_{ij} = a(t)^2 e^{2\Theta}\left(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{il}\gamma_{lj}\right) \qquad (9)$$

where, a(t) is the scale factor, Θ is the scalar perturbation, and γ is the tensor perturbation. The tensor perturbation is traceless ($\gamma_{ii} = 0$) and divergence-free ($\partial_i\gamma_{ij} = 0$), and is defined only up to the second order, since higher-order terms do not contribute to the action function. If we consider the decomposition theorem [18] for cosmological perturbations, each perturbation mode evolves separately, so each perturbation mode can be analyzed independently of each other.

The tensor perturbation γ_{ij} can be expressed in terms of two polarization modes,

$$\gamma_{ij} = h_+\widehat{\gamma_{ij}}^{+} + h_\times\widehat{\gamma_{ij}}^{\times} \qquad (10)$$

Considered in the Fourier space, $\widehat{\gamma_{ij}}^{+}$ and $\widehat{\gamma_{ij}}^{\times}$ satisfy the normalization conditions $\widehat{\gamma_{ij}}(\boldsymbol{k})\widehat{\gamma_{ij}}(-\boldsymbol{k})^* = 2$ and $\widehat{\gamma_{ij}}^{+}(\boldsymbol{k})\widehat{\gamma_{ij}}^{+}(-\boldsymbol{k})^* = 0$ [20].

Scalar Perturbation

The dynamic equation for each pertubation mode is obtained from the second order Lagrangian equation,

$$\mathcal{L}_2^{(s)} = a^3 Q_s \left[\dot{\Theta}^2 - \frac{c_s^2}{a^2}(\partial\Theta)^2\right] \quad (11)$$

$$Q_s \equiv \frac{2L_S(9\mathcal{W}^2 + 8L_S w)}{\mathcal{W}^2} \quad (12)$$

$$c_s^2 \equiv \frac{2}{Q_s}\left(\dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E}\right) \quad (13)$$

with,

$$L_S = \frac{1}{2}\left(1 - \zeta\phi^2 - \xi\frac{\dot{\phi}^2}{2}\right) \quad (14)$$

$$\mathcal{W} = 2\left[H - \zeta\left(H\phi^2 + \phi\dot{\phi}\right) - \frac{3\xi H\dot{\phi}^2}{2}\right] \quad (15)$$

$$w = 3\left[-9H^2 + 9\zeta\left(H^2\phi^2 + 2H\phi\dot{\phi}\right) + 27\xi H^2\dot{\phi}^2\right] \quad (16)$$

$$\mathcal{M} = \frac{4\zeta^2\phi^4 + 4\zeta\xi\phi^2\dot{\phi}^2 + \xi^2\dot{\phi}^4 + 16L_S - 4}{8\left[H\left(2L_S - \xi\dot{\phi}^2\right) - \xi\phi\dot{\phi}\right]} \quad (17)$$

$$\mathcal{E} = \frac{1}{2}\left[1 - \zeta\phi^2 + \frac{\xi\dot{\phi}^2}{2}\right] \quad (18)$$

In Fourier space, the equation of motion for a scalar perturbation (Θ) can be written in the form,

$$\ddot{\Theta} + \left(3H + \frac{\dot{Q}_s}{Q_s}\right)\dot{\Theta} + c_s^2\frac{k^2}{a^2}\Theta = 0 \quad (19)$$

To obtain the solution to equation (19), we consider the Bunch-Davies vacuum function,

$$\Theta(\tau,k) = \frac{iHe^{ic_s k\tau}}{2(2c_s k)^{3/2}\sqrt{Q_s}}(1 + ic_s k\tau) \quad (20)$$

On the superhorizon scale, $c_s k \ll aH$, the power spectrum is defined at $\tau \approx 0$ by,

$$\langle\Theta(0,\boldsymbol{k_1})\Theta(0,\boldsymbol{k_2})\rangle = \frac{2\pi^2}{k_1^3}P_\Theta(k_1)(2\pi)^3 \delta^{(3)}(\boldsymbol{k_1} + \boldsymbol{k_2}) \quad (21)$$

So if we consider the solution of Equation (20),

$$\Theta(0,k) = \frac{iH}{2(c_s k)^{3/2}\sqrt{Q_s}} \quad (22)$$

the power spectrum can be obtained,

$$P_\Theta = \frac{H^2}{8\pi^2 c_s^3 Q_s} \quad (23)$$

Furthermore, to characterize the fluctuations in the scalar perturbation power spectrum, the spectral index n_s is defined,

$$n_s - 1 \equiv \frac{d\ln P_\Theta}{d\ln k}\Big|_{c_s k = aH} = 2\frac{\dot{H}}{H} - \frac{\dot{Q}_s}{HQ_s} - 3\frac{\dot{c}_s}{Hc_s} \quad (24)$$

Since the perturbation evolves towards a constant on this scale, we will review the estimate of $c_s k = aH$ to evaluate the spectral index at inflation. The PLANCK collaboration data suggest that the scalar spectral index is limited to $n_s = 0.9665 \pm 0.0038$ at the 68% confidence level [5].

Tensor Perturbation

For tensor perturbation, the second-order Lagrangian equation has the form,

$$\mathcal{L}_2^{(t)} = \frac{a^3}{4}Q_t\left[\dot{\gamma}_{ij}^2 - \frac{c_t^2}{a^2}\left(\partial_k\gamma_{ij}\right)^2\right] \quad (25)$$

$$Q_t = \frac{1}{2}\left(1 - \zeta\phi^2 - \xi\frac{\dot{\phi}^2}{2}\right) \quad (26)$$

$$c_t^2 = \frac{1 - \zeta\phi^2 + \xi\frac{\dot{\phi}^2}{2}}{1 - \zeta\phi^2 - \xi\frac{\dot{\phi}^2}{2}} \quad (27)$$

Analogous to scalar perturbations, each polarization in the tensor perturbation $h_\lambda(\lambda = +,\times)$ satisfies the equation of motion,

$$\ddot{h}_\lambda + \left(3H + \frac{\dot{Q}_t}{Q_t}\right)\dot{h}_\lambda + c_t^2 \frac{k^2}{a^2} h_\lambda = 0 \quad (28)$$

Therefore, the solution for each polarization,

$$h_\lambda(\tau, k) = \frac{iHe^{-ic_t k\tau}}{2(2c_t k)^{\frac{3}{2}}\sqrt{Q_t}}(1 + ic_t k\tau) \quad (29)$$

so that the power spectrum for the tensor perturbation can be obtained,

$$P_h = \frac{H^2}{2\pi^2 Q_t c_t^3} \quad (30)$$

The factor of two arises because there are two polarizations of the tensor perturbation. The spectral index on $c_t k = aH$ is defined,

$$n_t \equiv \frac{d \ln P_\Theta}{d \ln k}|_{c_t k = aH} = 2\frac{\dot{H}}{H} - \frac{\dot{Q}_t}{HQ_t} - 3\frac{\dot{c}_t}{Hc_t} \quad (31)$$

Tensor-to-Scalar Ratio

The tensor-to-scalar ratio can be approximated via,

$$r \equiv \frac{P_h}{P_\Theta} = 4\frac{Q_s c_s^3}{Q_t c_t^3} \quad (32)$$

The PLANCK collaboration gave a value of $r < 0.10$ at the 95% confidence level.

COSMOLOGICAL PERTURBATION ANALYSIS

This study examines the de Sitter expansion, which causes the Hubble parameter to be constant,

$$a(t) \sim \exp(H_0 t) \rightarrow H(t) = \frac{\dot{a}}{a} = H_0 \quad (33)$$

The situation when the scalar field decays at large t is examined:

$$\phi \sim \exp(\phi_0 t) \quad (34)$$

$$\dot{\phi} \sim \phi_0 \exp(\phi_0 t) \quad (35)$$

$$\ddot{\phi} \sim \phi_0^2 \exp(\phi_0 t) \quad (36)$$

where, ϕ_0 must be negative to ensure scalar field decay. In this study, we will consider the special case where the minimal and non-minimal couplings are inversely related,

$$\zeta = \frac{1}{\xi} \quad (37)$$

After substituting the above conditions into the spectral index equation, we will plot the graphs for various values of ϕ_0. Furthermore, since it has been shown that inflation occurs when the universe undergoes exponential expansion beyond 60 e-folds [18], we consider $H_0 = 60$ and, based on background calculations [21], we consider the values $\zeta = 10^{-2}$, 10^{-3}, 10^{-4}, and 10^{-5}.

Scalar Perturbation

The spectral index of the scalar perturbation for the case plotted as a contour map as a function of time can be seen in Figure 1. As explained in the previous section, ϕ_0 must be negative and $H_0 > 60$ to ensure that the scalar field decays and inflation occurs. As seen in Figure 1, changes in the zeta value cause the curve to shift slightly toward higher times, indicating that the dynamics begin at different times. However, for each ζ, there is a value of ϕ_0 that can produce spectral index values close to scale invariance ($n_s \cong 1$), with slight differences.

Tensor Perturbation

Similar to the scalar perturbation, the spectral index of the scalar perturbation for the case plotted as a contour map as a function of time can be seen in Figure 2. As seen in Figure 2, changes in the zeta value do not significantly affect the spectral index values. However, for each ζ, there is a value of ϕ_0 that can provide spectral index values close to scale invariance ($n_t \cong 0$), with slight differences.

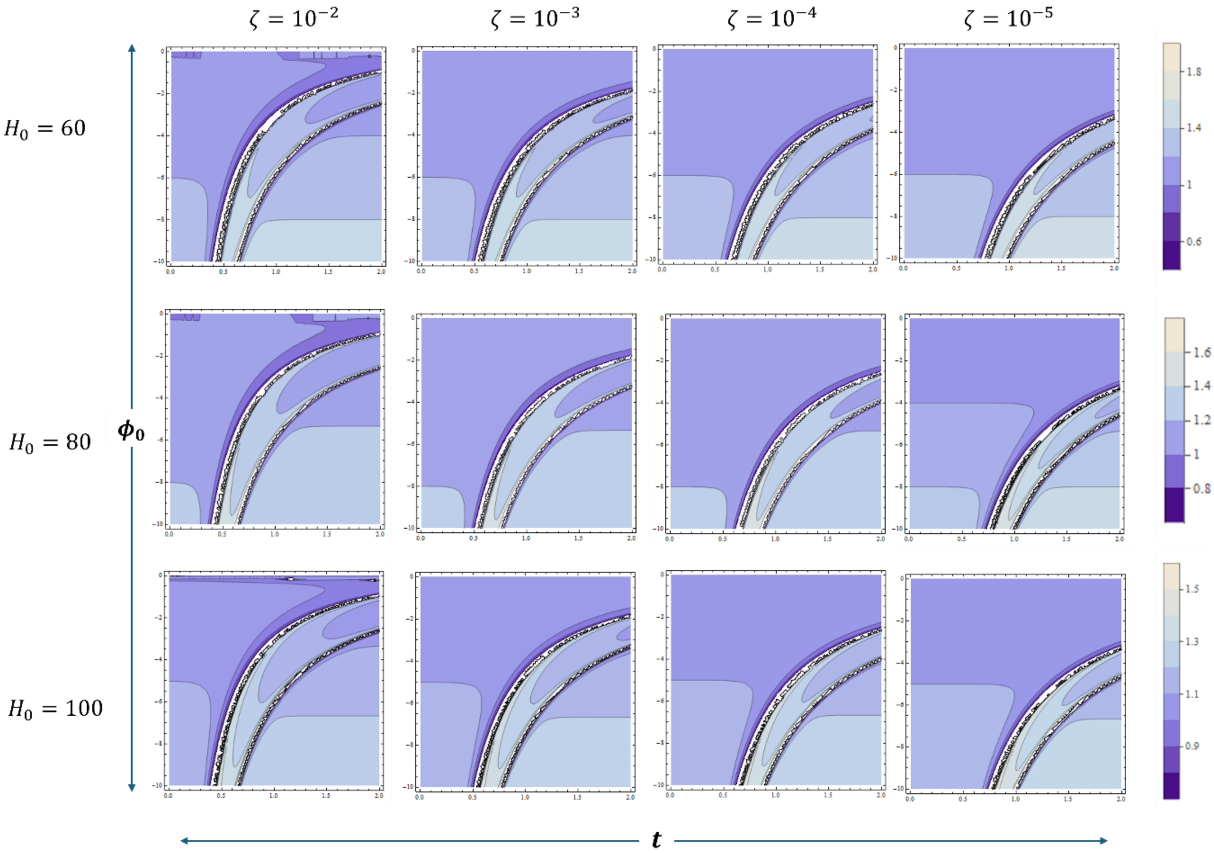

Figure 1. Contour plot of the scalar perturbation spectral index as a function of time and ϕ_0 for the cases H_0 = 60, 80 and 100 as well as $\zeta = 10^{-2}$, 10^{-3}, 10^{-4} and 10^{-5}.

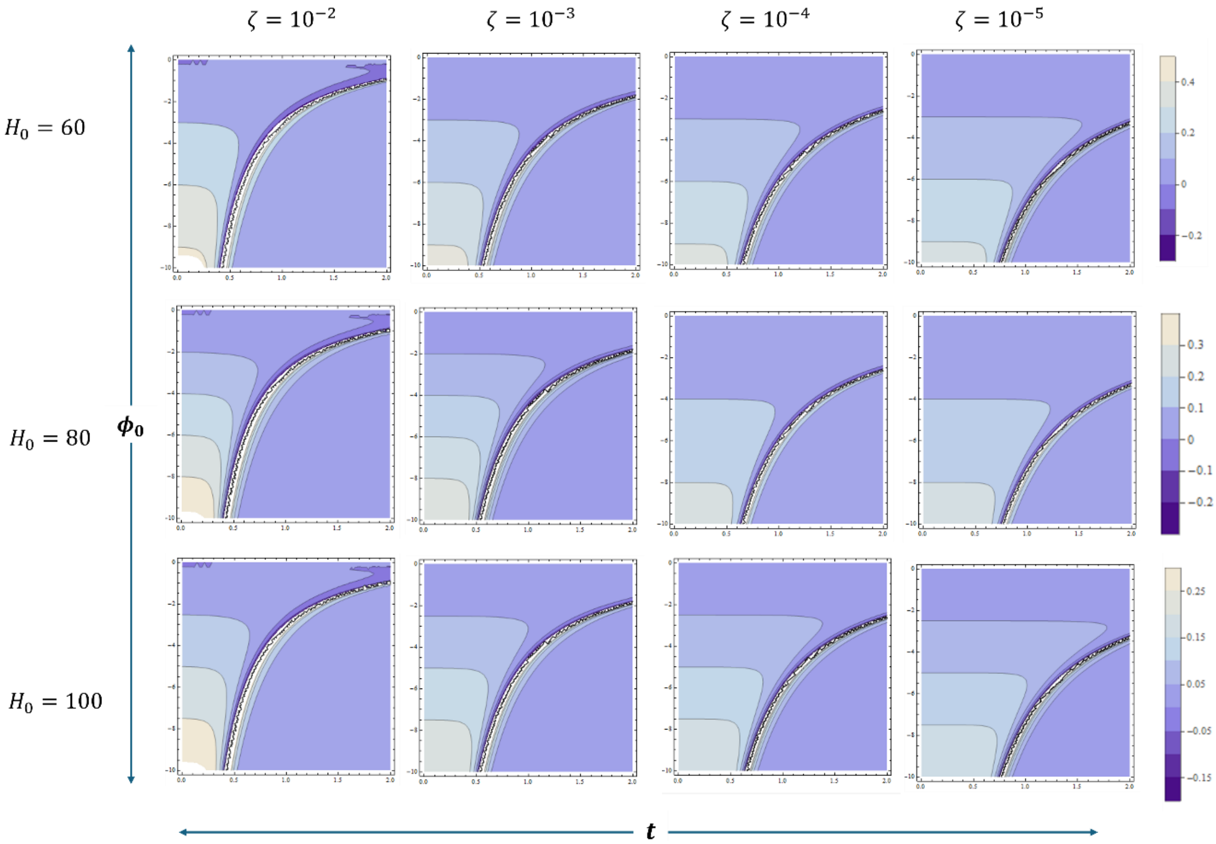

Figure 2. Contour plots of the spectral index perturbation tensor as a function of time and ϕ_0 for the cases H_0 = 60, 80 and 100 as well as $\zeta = 10^{-2}$, 10^{-3}, 10^{-4} and 10^{-5}.

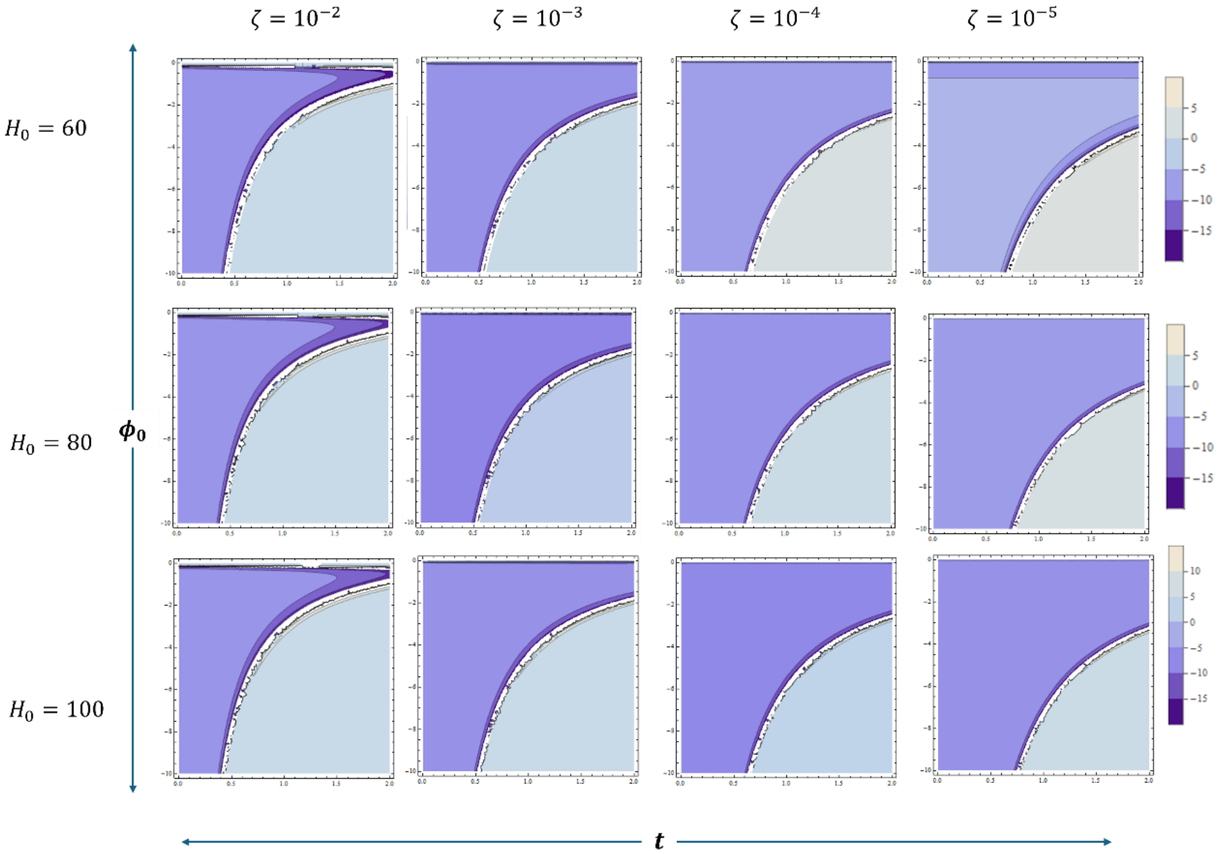

Figure 3. Contour plots of the tensor-to-scalar ratio as a function of time and ϕ_0 for the cases $H_0 = 60$, 80 and 100 as well as $\zeta = 10^{-2}$, 10^{-3}, 10^{-4} and 10^{-5}.

Tensor-to-Scalar Ratio

A plot of the tensor-to-scalar ratio as a function of time can be seen in Figure 3. The different ζ values exhibit similar behavior as for scalar and tensor perturbations. However, they still align with the observed data, with $r < 0.10$.

CONCLUSION

This study examines Horndeski's theory to construct a new cosmological inflationary model by selecting coefficient functions in the Horndeski Lagrangian that are free from the Ostrogradsky ghost. It also examines the Hubble parameter, which remains constant during inflation, and the scalar field decays as the inflationary period ends. Furthermore, the inverse relationship between the couplings, $\zeta=1/\xi$, is examined. By examining these approximations, spectral indices for scalar and tensor perturbations, as well as their tensor-to-scalar ratios, are derived and mapped for several values of the Hubble parameter (H_0) and the power of the scalar field (ϕ_0). As can be seen in Figures 1 and 2, the spectral indices are close to scale invariant and match the observational data. The same is true for the tensor-to-scalar ratio.

In this study, the system is "forced" to undergo inflation, considering the constant value of the Hubble parameter due to the complexity and nonlinear nature of the model under consideration. In future work, a more comprehensive analysis can be reviewed because it has the potential to provide good insights into cosmological inflation.

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