

# MULTI-PHOTONS TRAPPING STABILITY WITHIN A FIBER BRAGG GRATING FOR QUANTUM SENSOR USE

Saktioto

*Jurusan Fisika, FMIPA, University of Riau, Pekanbaru, 28293, Riau, Indonesia.*

## ABSTRACT

We propose an interesting result of the trapped multi photons distribution within a fiber Bragg grating. The trapped photons are confined by the potential well, which introduce the motion of photons in a fiber Bragg grating affected by multi perturbations. The external perturbations are defined as series of nonlinear parametric in terms of potential energy. This investigation is developed by using the nonlinear couple mode equations and under Bragg resonance condition where the initial frequency of the light,  $\omega_0$  is the same value as the Bragg frequency,  $\omega_B$ . The results show that the higher perturbation series represents the potential well is much indifferent of equilibrium. In applications, the perturbation can cause the trapped photons instability which introduces the escape photons from the potential well. The applications such as entangled photon source and quantum sensors can be performed.

**Keywords:** *Multi photon trapping; Bragg sensor; Quantum sensor; Entangled photon source, Quantum encoding*

## 1. INTRODUCTION

Fiber Bragg Grating (FBG) is known as a special device that when ultraviolet (UV) light is radiated on an optical fiber, the refractive index of the fiber core is changed due to the effect termed as photosensitivity [1]. FBG in optical fibers have been demonstrated in a wide range of applications such as for sensors, dispersion compensators, optical fiber filter and all optical switching and routing. The discovery of photosensitivity led to a wide range of range of optical fiber communication and optical sensor system. This process can be understood microscopically if we consider that light moves as particles. Therefore, numerous research is directed towards investigation of pulse propagation in FBGs [1-4]. Periodically structured optical media have been in the clarity of research activity for many years, due to versatile technologies applications in the fields of telecommunications and sensor system [1], and also as a subject of fundamental studies [2]. At the early

stage of the work in this area, the pioneering contribution by Winful *et. al.* [3] laid the groundwork for extensive theoretical activities exploring nonlinear pulse propagating in one dimensional periodic structure known as fiber Bragg gratings. They have considered the role of the Kerr nonlinearity in the light transmission through the FBGs. Bragg gratings in optical fibers are excellent devices for studying nonlinear phenomena particularly based on the Kerr nonlinearity [4]. These structures are based on the periodic modulation of the local periodic modulation of the local refractive index in the axial direction. A characteristic feature of FBGs is a stopband, alias photonic bandgap, in their linear-propagation spectrum. The bandgap is induced by the resonant coupling between the forward- and backward-propagating waves due to the Bragg resonance [5]. The stationary properties of one-dimensional Bragg gratings were first analyzed by Winful *et.al* [6]. Research on the existence of soliton

in FBGs has been reported [7]. Chen and Mills coined the term gap soliton in their numerical work covering the nonlinear optical super lattices [8]. Mills and Trullinger obtained an analytical solution for stationary gap solitary waves [9]. Sipe and Winful [10], Christoudolidis and Joseph [11], Aceves and Wabnitz [12], de Sterke and Sipe [13-14] and recently K. Senthilnathan *et. al* have derived the formation of bright and gap soliton solution for nonlinear coupled mode equation, which governs the pulse propagation in FBG [7].

The motion of a particle moving in FBG represents the pulse propagation in the grating structure of fiber optics exhibiting the existence of optical soliton. In order to describe the photon motion, the function of potential energy is depicted. Photon can be trapped by some parameters of potential energy such as  $\alpha$  and  $\gamma$ . The other parameter, theta,  $\theta$  is introduced to describe any disturbance effect of moving particle having specific energy.

In this paper we further describe the effect of  $\alpha$  and  $\gamma$  to obtain the optimized point of the potential well where the those parameters imposed to the energy stability.

## 2. MULTI PHOTONS POTENTIAL ENERGY DISTRIBUTION

Wave propagation in FBG is analyzed by solving Maxwell's equation with appropriate boundary conditions. In the presence of Kerr nonlinearity, using the coupled-mode theory, the nonlinear coupled mode equation is defined under the absence of material and waveguide dispersive effects. The dispersion arising from the periodic structure dominates near Bragg resonance conditions and it is valid only for wavelengths near to the Bragg wavelength. By substituting the stationary solution

to the coupled mode equation and by assuming  $E_{\pm}(z, t) = e_{\pm}(z)e^{-i\hat{\delta}ct/\bar{n}}$ , we obtain

$$\begin{aligned} i\frac{de_f}{dz} + \hat{\delta}e_f + \kappa e_b + (\Gamma_s |e_f|^2 + 2\Gamma_x |e_b|^2)e_f &= 0 \\ -i\frac{de_b}{dz} + \hat{\delta}e_b + \kappa e_f + (\Gamma_s |e_b|^2 + 2\Gamma_x |e_f|^2)e_b &= 0 \end{aligned} \quad (1)$$

Equation (1) represents the time-independent light transmission through the gratings structure where  $e_f$  and  $e_b$  are the forward and backward propagating modes [1]. In order to explain the formation of Bragg soliton, consider the Stokes parameter [11] since it will provide useful information about the total energy and energy difference between the forward and backward propagating modes.

$$\begin{aligned} A_0 &= |e_f|^2 + |e_b|^2, \\ A_1 &= e_f e_b^* + e_f^* e_b, \\ A_2 &= i(e_f e_b^* - e_f^* e_b), \\ A_3 &= |e_f|^2 - |e_b|^2 \end{aligned} \quad (2)$$

with the constraint  $A_0^2 = A_1^2 + A_2^2 + A_3^2$ . In the FBG theory, the nonlinear coupled-mode (NLCM) equation requires that the total power  $P_0 = A_3 = |e_f|^2 - |e_b|^2$  inside the grating is constant along the grating structures. Rewriting the NLCM equations in terms of Stokes parameter gives

$$\begin{aligned} \frac{dA_0}{dz} &= -2\kappa A_2, \quad \frac{dA_1}{dz} = 2\hat{\delta}A_2 + 3\Gamma A_0 A_2, \\ \frac{dA_2}{dz} &= -2\hat{\delta}A_1 - 2\kappa A_0 - 3\Gamma A_0 A_1, \quad \frac{dA_3}{dz} = 0 \end{aligned} \quad (3)$$

In Equation (3), we drop the distinction between the Self-Phase Modulation and cross effect modulation effects and hence it becomes  $3\Gamma = 2\Gamma_x + \Gamma_s$ . It can be clearly show that the total power,  $P_0 (=A_3)$  inside the grating is found to be constant or

conserved along the grating structure [2]. In the construction of the anharmonic oscillator type equation, it is necessary to use the conserved quantity, and it is obtained in the form

$$\hat{\delta}A_0 + \frac{3}{4}\Gamma A_0^2 + \kappa A_1 = C, \text{ where } C \text{ is the constant}$$

of integration and  $\hat{\delta}$  is the detuning parameter. Using Equation (3), we obtain

$$\frac{d^2 A_0}{dz^2} - \alpha A_0 + \beta A_0^2 + \gamma A_0^3 = 4\hat{\delta}C \quad (4)$$

where  $\alpha = 2[2\hat{\delta}^2 - 2\kappa^2 - 3\Gamma C]$ ,  $\beta = 9\Gamma\hat{\delta}$  and

$\gamma = \frac{9}{4}\Gamma^2$ . Equation (4) contains all the physical

parameter of the NLCM equation.

In order to describe the motion of a particle moving with the classic anharmonic potential, where the external disturbance is involved then we have the solution as follows,

$$V(A_0) = -\alpha \frac{A_0^2}{2} + \beta \frac{A_0^3}{3} + \gamma \frac{A_0^4}{4} \quad (5)$$

It represents the potential energy distribution in the Fiber Bragg Grating structures.

### 3. MULTI PHOTONS TRAPPING INSTABILITY

Consider Equation (2) having a set of constraints

which is introduced by  $\phi_{(e)} = \sum_{n=0}^{\infty} A_0^n$  as a function

of perturbation factor then

$$\frac{d^2 A_0}{dz^2} = \phi_e'' \Big|_{n=0} \quad (6)$$

If Equation (4) is accumulated using external perturbation then

$$\phi_e'' \Big|_{n=0} + \sum_{\substack{n=0 \\ m=1}}^{\infty} C_m A_0^n = \psi$$

where  $\psi$  is a function of  $f(\delta, C, C_m)$  and

$$C_m = [\alpha, \beta, \gamma, \dots]$$

The value of  $m = 2n$  for  $n = 1, 2, 3, \dots, m = 2n + 1$  for  $n = 0, 1, 2, \dots$

$C$  is constant and  $C = (C_1, C_2, C_3, \dots, C_m)$ . The value of  $C$  is linear to  $A_0$  but not to  $V$ . Equation (5) can then be modified by

$$V(A_0) = \sum_{\substack{m=1 \\ n=0}}^{\infty} C_m A_0^n \quad (7)$$

Equation (7) represents the complete potential energy distribution in the Fiber Bragg Grating structure. We believe at this juncture, the potential function is modified from Conti and Mills [14]. Using well-known Duffing oscillator type equation, analogically it is written as

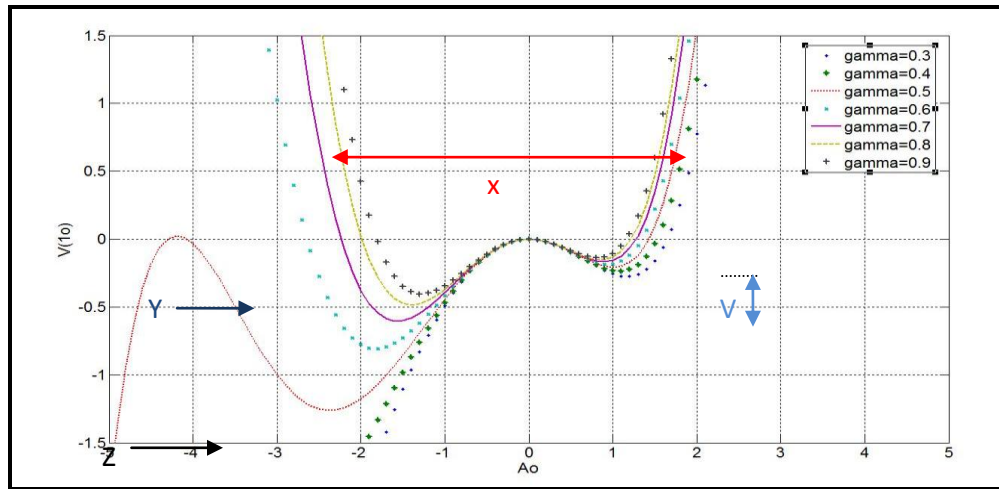
$$\phi_e'' + \sum_{\substack{m=1 \\ n=0}}^{\infty} C_m A_0^n = 0 \quad (8)$$

For multi perturbation of nonlinear parametric, two major shapes will be simplified in series term.

Fig. 1 depicts the motion of photon in potential well changes when few nonlinear parameter is take into account as shown is Equation (5), i.e. a diagram of launching light pulse into an FBG, the arrow for the applied external force to the grating region and at the output FBG-connect to the quantum processor . There are theoretically some comments in this figure. Photon is trapped by  $\alpha$  parameter which is depicted by legend V. When  $\alpha$  is too large, the potential well produces  $A_0$  an increase in and have a wider double well.  $\gamma$  parameter is shown by X legend. When  $\gamma$  is large, the potential well produces an increase in  $A_0$ . Suppose that the source that is imposed onto FBG than initial power is used to generate the particles. It shows that double well potential well is not symmetric and potential energy will decrease within the region at legend Y. The other effect is the perturbation of potential energy by

legend  $Z$  where photon cannot be trapped symmetrically. It will tend to equilibrate but it is not

stable where it will lead to losses.



**Fig. 1:** The motion of photon in potential well for  $\alpha = 0.9$ ,  $\beta = 0.3$ ,  $\theta = 0.09$  and  $\gamma$  is varies from 0.3 to 0.9.

In term of parametric function, we can describe it as follows. The change in  $\alpha$  will affect the dip of the potential well. If  $\alpha$  is approximately too small, the shape of the potential well will turn into a single potential well. The occurrence of  $\beta$  effect in the motion of photon will give effect to the negative region for  $A_0 < 0$ . The effect of  $\gamma$  shows that the width of potential well will decrease if we increased the value of  $\gamma$ . Therefore if we increased the value of  $\gamma$ , we can assume that the photon will be localized trapped. Another nonlinear factor,  $\theta$  will turns the shape of potential well rapidly. If we include the existence of  $\theta$ , the shape of potential well becomes chaotic. The photon does not only move in certain region known as potential well but also can be termed as free moving particles.

Fig. 2 explains the extrapolation of the graph if more factors of perturbation added into Equation (7). The addition of parametric factors by the higher odd number (Fig. 2(b)) will lead the photon to be untrapped and higher even number (Fig. 2 (a)) will allow the photon to move in a well. It is clearly shown in the graphs that as  $n \gg \infty$ , the value of  $|A_0|$  will remain constant in the range of  $-2 < A_0 < 2$ . However, when the value of  $V_{(0)}$  is equal to zero, there are many possibilities of  $A_0$ , meaning the exact value of intensity,  $A_0$  to trap the photon is difficult to determine in this condition. If the parametric factor is considered is too large then we may conclude that the photon is in indifferent state part of the equilibrium.

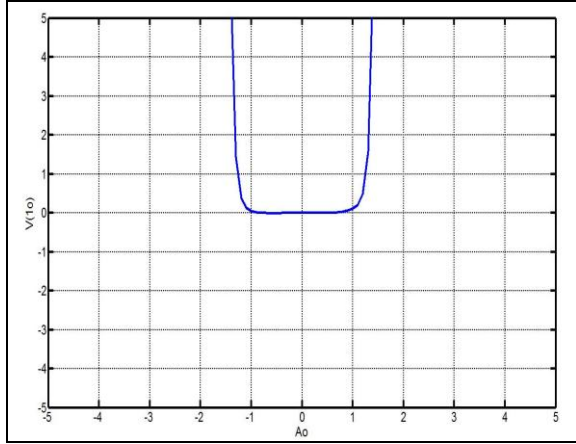


Fig. 2(a)

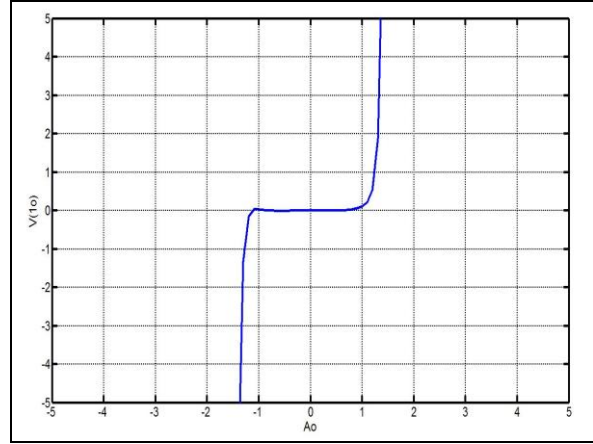


Fig. 2 (b)

**Fig.2:** The disturbance factor that affect the shape of the potential well of the motion of photon.

The stationary solutions of Equation (7) are applied neither for bright nor dark soliton solution since the dominant parameters in contributing  $A_0$  is unknown. However, from Equation (7) we have

$$A_0 = A_0(C_m, z) \quad (9)$$

Under these conditions, the frequencies with photonic band gap keep forming an envelope after the exact balancing at grating-induced dispersion with nonlinearity. It either decays or increased with the forward and backward waves being transferred by Bragg reflection process. The total energy of the system, potential energy function is equal to zero having multi perturbation which is  $-1 < A_0 < 1$  and if  $V \rightarrow \infty, A_0 = 2$ .

#### 4. PROPOSAL OF QUANTUM PROCESSING UNIT

Let us consider that the case when the photon output is input into the quantum processor unit. Generally, there are two pairs of possible polarization entangled photons forming within the ring device, which are represented by the four polarization orientation angles as  $[0^\circ, 90^\circ], [135^\circ \text{ and } 180^\circ]$ . These can be formed by using the optical component

called the polarization rotatable device and a polarizing beam splitter (PBS). In this concept, we assume that the polarized photon can be performed by using the proposed arrangement. Where each pair of the transmitted qubits can be randomly formed the entangled photon pairs. To begin this concept, we introduce the technique that can be used to create the entangled photon pair (qubits) as shown in Fig. 15, a polarization coupler that separates the basic vertical and horizontal polarization states corresponds to an optical switch between the short and the long pulses. We assume those horizontally polarized pulses with a temporal separation of  $\Delta t$ . The coherence time of the consecutive pulses is larger than  $\Delta t$ . Then the following state is created by Eq. (6) [11,12].

$$|\Phi\rangle_p = |1, H\rangle_s |1, H\rangle_i + |2, H\rangle_s |2, H\rangle_i \quad (10)$$

In the expression  $|k, H\rangle, k$  is the number of time slots (1 or 2), where denotes the state of polarization [horizontal  $|H\rangle$  or vertical  $|V\rangle$ ], and the subscript identifies whether the state is the signal (s) or the idler (i) state. In Eq. (6), for simplicity, we have omitted an amplitude term that is common to all product

states. We employ the same simplification in subsequent equations in this paper. This two-photon state with  $|H\rangle$  polarization shown by Eq. (6) is input into the orthogonal polarization-delay circuit shown schematically. The delay circuit consists of a coupler and the difference between the round-trip times of the micro ring resonator, which is equal to  $\Delta t$ . The micro ring is tilted by changing the round trip of the ring is converted into  $|V\rangle$  at the delay circuit output. That is the delay circuits convert  $|k, H\rangle$  to be

$$r|k, H\rangle_s + t_2 \exp(i\phi)|k+1, V\rangle_s + r t_2 \exp(i_2\phi)|k+2, H\rangle_s + r_2 t_2 \exp(i_3\phi)|k+3, V\rangle_s$$

Where  $t$  and  $r$  is the amplitude transmittances to cross and bar ports in a coupler. Then Eq. (6) is converted into the polarized state by the delay circuit as

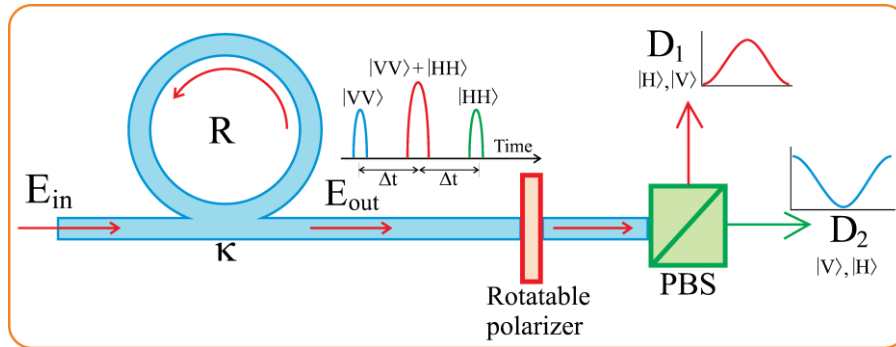
$$\begin{aligned} |\Phi\rangle &= [|1, H\rangle_s + \exp(i\phi_s)|2, V\rangle_s] \times [|1, H\rangle_i + \exp(i\phi_i)|2, V\rangle_i] \\ &+ [|2, H\rangle_s + \exp(i\phi_s)|3, V\rangle_s] \times [|2, H\rangle_i + \exp(i\phi_i)|2, V\rangle_i] \\ &= [|1, H\rangle_s |1, H\rangle_i + \exp(i\phi_i)|1, H\rangle_s |2, V\rangle_i] \\ &+ \exp(i\phi_s)|2, V\rangle_s |1, H\rangle_i + \exp[i(\phi_s + \phi_i)]|2, V\rangle_s |2, V\rangle_i \\ &+ |2, H\rangle_s |2, H\rangle_i + \exp(i\phi_i)|2, H\rangle_s |3, V\rangle_i \end{aligned}$$

$$+ \exp(i\phi_s)|3, V\rangle_s |2, H\rangle_i + \exp[i(\phi_s + \phi_i)]|3, V\rangle_s |3, V\rangle_i \quad (11)$$

By the coincidence counts in the second time slot, we can extract the fourth and fifth terms. As a result, we can obtain the following polarization entangled state as

$$\begin{aligned} |\Phi\rangle &= |2, H\rangle_s |2, H\rangle_i \\ &+ \exp[i(\phi_s + \phi_i)]|2, V\rangle_s |2, V\rangle_i \quad (12) \end{aligned}$$

We assume that the response time of the Kerr effect is much less than the cavity round-trip time. Because of the Kerr nonlinearity of the optical device, the strong pulses acquire an intensity dependent phase shift during propagation. The interference of light pulses at a coupler introduces the output beam, which is entangled. Due to the polarization states of light pulses are changed and converted while circulating in the delay circuit, where the polarization entangled photon pairs can be generated. The entangled photons of the nonlinear ring resonator are separated to be the signal and idler photon probability. The polarization angle adjustment device is applied to investigate the orientation and optical output intensity, this concept is well described by the published work [13].



**Fig. 3.** A schematic of an entangled photon pair manipulation within a ring resonator. The Bell's state is propagating to a rotatable polarizer and then is split by a beam splitter (PBS) flying to detector  $D_1$  and  $D_2$ .

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Figure 3 is underway to generate and compare to the perturbation condition for energy stability.

## 5. CONCLUSION

We successfully modified and developed the potential energy distribution of photon by setting the disturbance of multi perturbation potential energy in a fiber Bragg grating. It is found that the potential well under Bragg resonance condition is not symmetrical and conserved. The higher perturbation series representing the potential well is much indifferent of the equilibrium in both odd and even nonlinear parametric factor of  $n$ .

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